Contributions of leptoquark interactions into the tensor and scalar form factors of $K^+ \to \pi^0 l^+ \nu_l$ decay

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Abstract

In the framework of scalar-vector dominance we calculate the hadronic matrix elements of scalar and tensor effective quark currents induced by virtual leptoquark interactions. Combined bounds on the product of couplings and leptoquark masses are obtained from experimental data.

1 Introduction

Recently, new data on the analysis of $K^+ \to \pi^0 l^+ \nu_l$ decays were published by two collaborations: KEK-E246 [1] and ISTRA [2], in addition to the presentation given by the Particle Data Group [3]. So, at present we have got quite precise measurements of characteristics in the K_{l3} decays, that needs a theoretical interpretation in the framework of Standard Model (SM) as well as beyond it. Such the study is of interest because of the experimental search for effects, which can point to the contributions with the violation of combined CP-parity in the kaon decays, for example, the transverse T-odd polarization of lepton in $K_{l3\gamma}$ modes [4], that can essentially enrich the information on the CP-breaking dynamics in addition to the program with the B-mesons [5].

The matrix element of decay is parameterized in terms of scalar, vector and tensor form factors, f_S, $f_\pm$ and f_T, in the following general form [6]:

$$\mathcal{M}[K^+ \to \pi^0 l^+ \nu_l] = G_{\rm F} V_{su} \left[-l_\mu (f_+ p^\mu + f_- q^\mu) + 2 m_K l_S f_S + {\rm i}\, \frac{f_T}{m_K}\, l_{\mu\nu} p^\mu q^\nu \right], \tag{1}$$

where the lepton currents are given by the expressions

$$\begin{aligned} l_\mu &= \bar\nu_{\rm L} \gamma_\mu l_{\rm L}, \\ l_{\mu\nu} &= \bar\nu_{\rm L} \sigma_{\mu\nu} l_{\rm R}, \\ l_S &= \bar\nu_{\rm L} l_{\rm R}, \end{aligned}$$

so that chiral spinors are

$$\theta_{\rm R,L} = \frac{1}{2}(1 \pm \gamma_5)\, \theta,$$

and $G_{\rm F}$ is the Fermi constant, m_K is the mass of kaon. The four-momenta are defined as

$$p = p_K + p_\pi, \quad q = p_K - p_\pi,$$

while V_{su} is the matrix element of Cabibbo–Kobayashi–Maskawa matrix for the mixing of weak charged quark-currents. We define the generators σ by the commutator

$$\sigma_{\mu\nu} = \frac{\mathrm{i}}{2}[\gamma_\mu, \gamma_\nu].$$

The dependence of form factors on q^2 is usually expressed in terms of linear slopes normalized to get the dimensionless quantities

$$\lambda_i = \left.\frac{\mathrm{d}\ln f_i(q^2)}{\mathrm{d}q^2/m_\pi^2}\right|_{q^2=0}, \tag{2}$$

where m_π is the pion mass. The combination of form factors

$$f_0 = f_+ + \frac{q^2}{p\cdot q}\, f_-, \tag{3}$$

is introduced, so that the experimental data are given in terms of the following set [6]:

$$\lambda_+, \quad \lambda_0, \quad \frac{f_{S,T}}{f_+(0)}.$$

The data of [1, 2] on $\lambda_{+,0}$ can be averaged, so that with the statistical errors we get

$$\lambda_+ = 0.0287 \pm 0.0018, \tag{4}$$
$$\lambda_0 = 0.0203 \pm 0.0033, \tag{5}$$

while the systematic uncertainties are given in the original papers. The values in (4) and (5) result in the ratio

$$\frac{f_-(0)}{f_+(0)} = -0.096 \pm 0.043. \tag{6}$$

The given parameter λ_+ is in a good agreement with the PDG values for both the electron and muon modes [3], while λ_0 and $f_-(0)/f_+(0)$ above are within the limits of $1.5\,\sigma$-deviations from the PDG averages. The preliminary analysis by KTeV [7] gives

$$\lambda_+ = 0.0275 \pm 0.0008,$$

which is close to the estimate in (4).

In the framework of SM we get the form factors

$$\langle \pi^0(p_\pi)|\,\bar{s}\gamma_\mu u\,|K^+(p_K)\rangle = \frac{1}{\sqrt{2}}\,(f_+ p_\mu + f_- q_\mu), \tag{7}$$

while

$$f_S = f_T = 0.$$

Therefore, the study of scalar and tensor form factors is a good test for the search of ‘new’ physics beyond the SM.

Supposing (7), we derive

$$q^\mu \langle \pi^0(p_\pi)|\, \bar{s}\gamma_\mu u\, |K^+(p_K)\rangle = (m_u - m_s)\, \langle \pi^0(p_\pi)|\, \bar{s} u\, |K^+(p_K)\rangle = (p \cdot q)\, \frac{f_0}{\sqrt{2}}, \tag{8}$$

implying that the form factor f_0 determines the matrix element of scalar quark-current.

The contraction of vector lepton-current

$$-l_\mu\, q^\mu\, f_- = m_l f_-\, l_S,$$

induces the scalar term in the matrix element. So, the electron mode is more sensitive to the extraction of scalar form factor, since the SM background contribution is suppressed by the lepton mass,

$$f_S^{\rm SM} = \frac{m_l}{2m_K}\, f_-.$$

At present, the measurements of f_S and f_T result in values slightly deviating from zero, that is consistent with the expectations of SM. So, in the electron mode [1]

$$\frac{f_S}{f_+(0)} = 0.0040 \pm 0.0160(\text{stat.}) \pm 0.0067(\text{syst.}), \tag{9}$$

$$\frac{f_T}{f_+(0)} = -0.019 \pm 0.080(\text{stat.}) \pm 0.038(\text{syst.}), \tag{10}$$

where we have taken into account the redefinition of sign in comparison with the appropriate formula in [1] as accepted in this paper in (1), while the combined analysis of muon and electron modes in [2] results in the similar values

$$\frac{f_S}{f_+(0)} = 0.004 \pm 0.005(\text{stat.}) \pm 0.005(\text{syst.}), \tag{11}$$

$$\frac{f_T}{f_+(0)} = -0.021 \pm 0.028(\text{stat.}) \pm 0.014(\text{syst.}). \tag{12}$$

The collaboration KTeV presented the following constraints in the electron mode:

$$\left|\frac{f_S}{f_+(0)}\right| < 0.04, \tag{13}$$

$$\left|\frac{f_T}{f_+(0)}\right| < 0.14. \tag{14}$$

In the present paper we study nonzero contribution of leptoquark interactions to the tensor form factor, which correlates with the scalar one due to the Fierz transformation. In section 2 the effective lagrangians with the virtual leptoquarks are described as concerns for the decays of $K^+ \to \pi^0 l^+ \nu_l$, and the requared matrix elements of quark currents are presented. General expressions for the hadronic matrix elements with the tensor structure are derived in section 3, where we develop the model based on the dominance of vector and scalar mesons and adjust it in the description of $f_{\pm,0}$ form factors. In the framework of potential approach the preferable region of model parameter is limited in agreement with the experimental data. The constraints on the masses of scalar leptoquark and their couplings to the fermions are obtained in section 4. The results are summarized in the Conclusion.

2 The contribution of leptoquark interactions

A consistent classification of leptoquarks under the gauge symmetries of SM were done by Buchmüller, Rückl and Wyler in [8]. We accept the nomenclature prescribed in [9] as shown in Table 1 extracted from [10]. So, the leptoquarks are marked by their spin, representation of weak $SU(2)$-group (singlets, doublets and triplets), appropriate electric charges in the multiplets and the fermion number F. For the sake of briefness, the flavor of lepton is marked by the electron in Table 1, while the couplings $\mathrm{Y}_{\mathrm{L,R}}$ should be labelled by the flavor indices, too.

The diagrams describing the contribution of leptoquark interactions into the form factors under study are shoWn in Fig. 1.

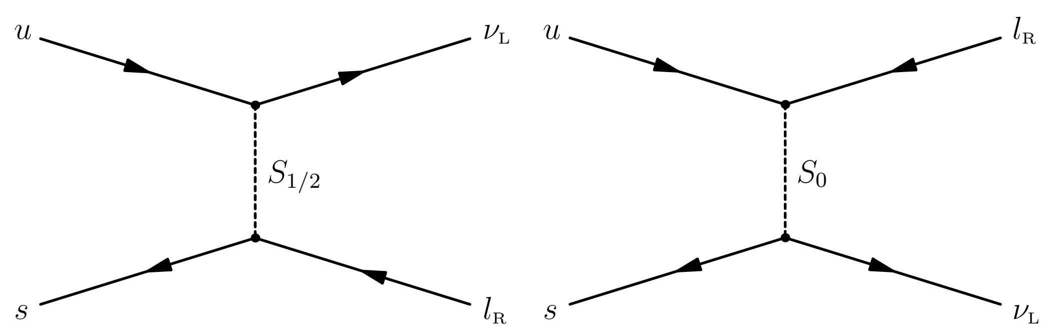

Figure 1: Two kinds of leptoquark exchanges contributing to the tensor form factor in the decay $K^+ \to \pi^0 l^+ \nu_l$.

The tensor terms appear under the Fierz transformations, so that the vector leptoquarks do not contribute into the tensor form factor. Further, the tensor term shifts the helicity of leptons. Therefore, we isolate the leptoquarks involving the interaction with both the left-handed neutrinos and right-handed charged leptons. The appropriate vertices are shaded in Table 1. Thus, we consider the following scalar leptoquarks: the singlet S_0 and the doublet $S_{1/2}$ with the charge $-2/3$.

The Yukawa-like interactions involving the strange quark have the form

$$\begin{aligned} \mathcal{L}[S_{1/2}] &= S^*_{1/2}\,(\mathrm{Y}_{\mathrm{L}}\,\bar{u}_{\mathrm{R}}\nu_{\mathrm{L}} + \mathrm{Y}_{\mathrm{R}}\,\bar{s}_{\mathrm{L}} l_{\mathrm{R}}) + \text{h.c.}, && (15)\\ \mathcal{L}[S_0] &= S^*_0\,[\mathrm{Y}^{[0]}_{\mathrm{L}}\,(\bar{u}_{\mathrm{C,R}} e_{\mathrm{L}} + \bar{s}_{\mathrm{C,R}}\nu_{\mathrm{L}}) + \mathrm{Y}^{[0]}_{\mathrm{R}}\,\bar{u}_{\mathrm{C,L}} l_{\mathrm{R}}] + \text{h.c.}, && (16)\end{aligned}$$

where we have omitted the flavor indices. These lagrangians induce the effective low-energy interactions according to the formulae

$$\begin{aligned} \mathcal{L}_{\text{eff}} &= -\frac{1}{8}\,\frac{\mathrm{Y}_{\mathrm{R}}\mathrm{Y}^*_{\mathrm{L}}}{M^2_{S_{1/2}}}\,(\bar{s}_{\mathrm{L}}\sigma_{\alpha\beta}u_{\mathrm{R}})\,(\bar{\nu}_{\mathrm{L}}\sigma^{\alpha\beta}l_{\mathrm{R}}) - \\ &\quad -\frac{1}{2}\,\frac{\mathrm{Y}_{\mathrm{R}}\mathrm{Y}^*_{\mathrm{L}}}{M^2_{S_{1/2}}}\,(\bar{s}_{\mathrm{L}}u_{\mathrm{R}})\,(\bar{\nu}_{\mathrm{L}}l_{\mathrm{R}}) + \text{h.c.}, && (17)\end{aligned}$$

scalar LQ ($\tilde{q}$)	charge	F	coupling and decay mode	β_e
S_0 (or $\tilde{d}_R$)	$-1/3$	2	Y_L : $e^-_L u$, $\nu_L d$	1/2
			Y_R : $e^-_R u$	1
$\tilde{S}_0$	$-4/3$	2	Y_R: $e^-_R d$	1
$\tilde{S}_{1/2}$ (or $\tilde{d}_L$)	$+1/3$	0	Y_L: $\nu_L \bar{d}$	0
$\tilde{S}_{1/2}$ (or $\bar{\tilde{u}}_L$)	$-2/3$	0	Y_L: $e^-_L \bar{d}$	1
S_1	$+2/3$	2	Y_L : $\nu_L u$	0
	$-1/3$		Y_L : $\nu_L d$, $e^-_L u$	1/2
	$-4/3$		Y_L : $e^-_L d$	1
$S_{1/2}$	$-2/3$	0	Y_L : $\nu_L \bar{u}$	0
			Y_R : $e^-_R \bar{d}$	1
	$-5/3$		Y_L : $e^-_L \bar{u}$	1
			Y_R : $e^-_R \bar{u}$	1

vector LQ	charge	F	coupling and decay mode	β_e
$V_{1/2}$	$-1/3$	2	Y_L : $\nu_L d$	0
			Y_R : $e^-_R u$	1
	$-4/3$		Y_L : $e^-_L d$	1
			Y_R : $e^-_R d$	1
$\tilde{V}_{1/2}$	$+2/3$	2	Y_L : $\nu_L u$	0
	$-1/3$		Y_L : $e^-_L u$	1
V_0	$-2/3$	0	Y_L : $e^-_L \bar{d}$, $\nu_L \bar{u}$	1/2
			Y_R : $e^-_R \bar{d}$	1
V_1	$+1/3$	0	Y_L : $\nu_L \bar{d}$	0
	$-2/3$		Y_L : $e^-_L \bar{d}$, $\nu_L \bar{u}$	1/2
	$-5/3$		Y_L : $e^-_L \bar{u}$	1
$\tilde{V}_0$	$-5/3$	0	Y_R: $e^-_R \bar{u}$	1

Table 1: The first generation scalar (S) leptoquarks/squarks and vector (V) leptoquarks in the BRW model [8] according to the nomenclature in [9] with their electric charge in units of e and fermion number $F = L + 3B$. For each possible non-zero coupling Y the decay modes and the corresponding branching ratio β_e for the decay into an electron and a quark are also listed. The restrictions on the values of β_e arise from the assumption of chiral couplings.

where we have used the Fierz transformations for the chiral fermions, taking into account the identity

$$\gamma_5\, \theta_{\rm R} = \theta_{\rm R},$$

that causes the summation of scalar and pseudoscalar parts (the factor of 2). The anti-commutation of fermions has been explored, too (the overall negative sign). Further we introduce the notation

$$\frac{1}{\Lambda^2_{LQ}} = \frac{{\rm Y}_{\rm R}{\rm Y}^*_{\rm L}}{M^2_{S_{1/2}}}, \tag{18}$$

since the above combination of leptoquark mass and couplings enters the problem under study.

As for the contribution of S_0, one can easily find that the effective lagrangian has the same form of (17), because the charge conjugation of spinor is defined by

$$\theta_{\rm C} = {\rm C}\,\theta^*,$$

where ${\rm C} = {\rm i}\gamma_2$ in the Dirac representation of γ-matrices, so that

$${\rm C}\gamma_\mu^{\rm T}{\rm C}^{-1} = -\gamma_\mu,$$

where T denotes the transposition. The terms induced by the leptoquarks $S_{1/2}$ and S_0 can interfere, of course. However, we include this effect into the definition of scale Λ_{LQ}.

Thus, we can estimate the contribution of leptoquark interactions, once we calculate the appropriate matrix elements of quark currents, that is the deal of next section.

3 Hadronic matrix elements

The experimental data on the slopes of form factors shown in the Introduction are in a good agreement with the estimates in the framework of chiral perturbation theory (χPT) [11]. However, to the moment we have not any predictions of χPT on hands as concerns for the hadronic matrix elements of tensor quark-current. In the present paper we explore the model of meson dominance, i.e. the dominance of vector and scalar states appropriate for the quantum numbers of transitions between the quarks. The corresponding diagram is shown in Fig. 2.

Considering the vector quark-current, we can evaluate the form factors

$$f_+(q^2) = g_{K^*K\pi}\frac{f_{K^*}\, m_{K^*}}{m^2_{su}(q^2)}\,\frac{1}{1-q^2/m^2_{su}(q^2)}, \tag{19}$$

$$f_-(q^2) = -f_+(q^2)\frac{m_K^2-m_\pi^2}{m^2_{su}(q^2)} + g_{K_0^*K\pi}\frac{f_{K_0^*}}{m^2_{K_0^*}}\,\frac{1}{1-q^2/m^2_{K_0^*}}, \tag{20}$$

in terms of couplings entering the following Lagrangians[1]

$$\mathcal{L}_{K^*K\pi} = g_{K^*K\pi}\,(p_K+p_\pi)_\mu\,\epsilon^\mu_{K^*}\,\varphi^*_K\,\varphi^*_\pi, \tag{21}$$

$$\mathcal{L}_{K_0^*K\pi} = g_{K_0^*K\pi}\,\varphi_{K_0^*}\,\varphi^*_K\,\varphi^*_\pi, \tag{22}$$

[1]The couplings g are prescribed for the charged pions, while the neutral ones have the isospin factor $1/\sqrt{2}$.

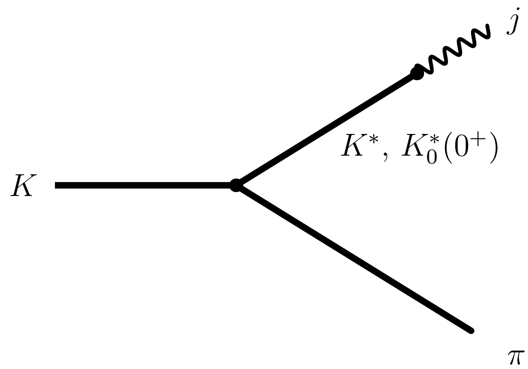

Figure 2: The diagram describing the contribution of excited vector and scalar kaon states into the hadronic matrix element of current j factorized from the lepton part in the decay $K^+ \to \pi^0 l^+ \nu_l$.

and

$$\langle K^*(k)|\, \bar{s}\gamma_\mu u\, |0\rangle \;=\; f_{K^*}\, \epsilon_\mu^{K^*}\, m_{K^*}, \tag{23}$$

$$\langle K_0^*(k)|\, \bar{s}\gamma_\mu u\, |0\rangle \;=\; f_{K_0^*}\, k_\mu, \tag{24}$$

where φ denotes the appropriate field, and ϵ is the polarization vector of K^*. In (19) we have introduced the running pole mass $m_{su}(q^2)$ in the transition $s \to u$. The normalization condition is rather evident

$$m_{su}(m_{K^*}^2) = m_{K^*},$$

while we need the value at $q^2 = 0$, $m_{su} = m_{su}(0)$, since we use the approximation of linear evolution of form factors,

$$f_+(q^2) \;\approx\; g_{K^* K\pi} \frac{f_{K^*}\, m_{K^*}}{m_{su}^2}\, (1 + q^2/m_{su}^2), \tag{25}$$

$$f_-(q^2) \;\approx\; -f_+(q^2) \frac{m_K^2 - m_\pi^2}{m_{su}^2} + g_{K_0^* K\pi} \frac{f_{K_0^*}}{m_{K_0^*}^2}\, (1 + q^2/m_{K_0^*}^2). \tag{26}$$

The evolution of $m_{su}(q^2)$ to $q^2 = 0$ is expected to be slow in the framework of model with the meson dominance. We suppose that the spin forces in the bound state should be suppressed beyond the pole, since they depend on a density of bound states, which drops outside the poles. So, the spin-averaged mass of 1S-level in the $\bar{s}u$ system is known experimentally,

$$m_{su}[1S] = \frac{1}{4}(m_K + 3 m_{K^*}) \approx 793 \text{ MeV}.$$

We expect that

$$m_{su}[1S] < m_{su}(0) < m_{K^*}.$$

So, we put

$$m_{su} \approx \frac{1}{2}(m_{su}[1S] + m_{K^*}) \approx 0.85 \text{ GeV}, \tag{27}$$

which is inside the systematics uncertainty of the model. Since the spin-dependent forces are suppressed in the excited P-waves, we put the pole mass in the scalar sector to be equal to the experimental value of K_0^*.

From (25) and (26) one can easily deduce the expression for the scalar-channel form factor,

$$f_0(q^2) \approx f_+(0) + q^2 \frac{g_{K_0^* K\pi} f_{K_0^*}}{m^2_{K_0^*}(m_K^2 - m_\pi^2)}, \tag{28}$$

as well as the slopes,

$$\lambda_+ = \frac{m_\pi^2}{m_{su}^2}, \tag{29}$$

$$\lambda_0 = \delta \cdot \lambda_+, \tag{30}$$

where

$$\delta = \frac{1}{f_+(0)} \frac{g_{K_0^* K\pi} f_{K_0^*} m_{su}^2}{m^2_{K_0^*}(m_K^2 - m_\pi^2)}, \tag{31}$$

$$f_+(0) = g_{K^* K\pi} \frac{f_{K^*}\, m_{K^*}}{m_{su}^2}. \tag{32}$$

The most of model parameters can be extracted from the experimental data. So, the coupling constant f_{K^*} is well known,

$$f_{K^*} \approx 215 \text{ MeV},$$

while the decay constants g are related with the widths measured[2],

$$\Gamma[K^* \to K\pi] = g^2_{K^* K\pi} \frac{|\boldsymbol{p}_K|^3}{4\pi m^2_{K^*}}, \tag{33}$$

$$\Gamma[K_0^* \to K\pi] = g^2_{K_0^* K\pi} \frac{3\,|\boldsymbol{p}_K|}{16\pi m^2_{K_0^*}}, \tag{34}$$

whereas $|\boldsymbol{p}_K|$ denotes the momentum of kaon in the c.m.s, so that numerically[3]

$$g_{K^* K\pi} \approx 3.94, \quad g_{K_0^* K\pi} \approx 3.48 \text{ GeV}.$$

[2]In the formulae for the total widths of K^* and K_0^* we have explored the isospin-symmetry relations: $\Gamma[K^{*+} \to K^+\pi^0] = 1/2\,\Gamma[K^{*+} \to K^0\pi^+]$ and the similar equation for the scalar meson K_0^*.

[3]In the estimates we put the effective masses in the equations relating the constants with the total widths, so that $m_{K^*} \to m_{su}[1S]$ and $m_{K_0^*} \to 2|\boldsymbol{p}_K|$ in the limit of $m_{K_0^*} \gg m_K, m_\pi$. In the phenomenological model under study, the decay constants g enter the form factors in terms of products with the leptonic couplings f. These products should be adjusted in order to satisfy some conditions motivated by QCD and its chiral symmetry. In this way we have to follow a specified approach in estimates for both g and f as given below. We stress that the model parameters g are quite uncertain because of reasons inherent for the phenomenological approach ignoring higher excitations as well as a continuum contribution. Neverteless, we argue for the preerable choice of numerical values.

The only free parameter of the model is the coupling $f_{K_0^*}$, which we are tending to restrict in the framework of potential calculations by the comparative analysis with the known leptonic constants of ρ and K^*. For this purpose, we calculate the diagram in Fig. 3, where the quark-meson vertex includes the wave function of constituent quarks.

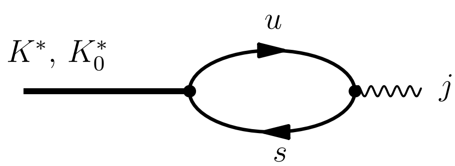

Figure 3: The diagram describing the contribution of quark loop into the hadronic matrix element of current j.

For the scalar state we use the current

$$j(x) = \bar{s}(x)u(x),$$

with the identity

$$\mathrm{i}\partial_\mu\left[\bar{s}(x)\gamma^\mu u(x)\right] = (m_u - m_s)j(x).$$

In this technique we find

$$f_{K_0^*}^{\mathrm{PM}} = \frac{m_s - m_u}{m_{K_0^*}}\frac{18}{m_{\mathrm{red}}\sqrt{\pi m_{K_0^*}}}\,|R'_{\bar{s}u}(0)|, \tag{35}$$

$$f_{K^*}^{\mathrm{PM}} = \sqrt{\frac{3}{\pi m_{\bar{s}u}[1S]}}\,|R_{\bar{s}u}(0)|, \tag{36}$$

where $R_{\bar{s}u}(r)$ denotes the radial wave function in the system of $\bar{s}u$, $m_{\bar{s}u}[1S]$ is the spin-averaged mass of K^* and K, and m_{red} is the constituent reduced mass for K_0^*, so that

$$m_{\mathrm{red}} \approx \frac{m_{\bar{d}u}[1S]m_{\bar{s}u}[1S]}{m_{\bar{d}u}[1S] + m_{\bar{s}u}[1S]} \approx 0.34\ \mathrm{GeV}.$$

For the ρ meson we have the expression similar to (36) under the substitution $\bar{s} \to \bar{d}$.

Further, we explore the static potential derived in [12] and solve the Schrödinger equation

$$\left[\frac{\boldsymbol{p}^2}{\mu_q} + V(r)\right]\Psi(r) = [\bar{\Lambda}(\mu_q) + 2(\mu_0 - \mu_q)]\Psi(r), \tag{37}$$

for the system $\bar{d}u$, so that the binding energy $\bar{\Lambda}(\mu_q)$ of $1S$-level is related with the mass

$$m_{\bar{d}u}[1S] = \bar{\Lambda}(\mu_q) + 2\delta\mu, \tag{38}$$

and it is shown in Fig. 4 at $\mu_0 = 0.345$ GeV, $\mu_q^\star = 0.224$ GeV versus the light quark constituent mass μ_q with $\delta\mu = \mu_q^\star - \mu_0$. In (38) we do not add the constituent masses of light quarks into

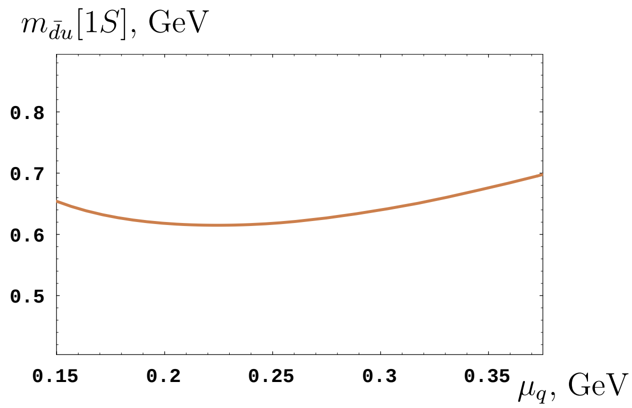

Figure 4: The mass of $1S$ level in the system $\bar{d}u$ calculated in the potential model with the constituent mass μ_q.

the mass of meson, since the constituent masses are really the parts of potential energy $V(r)$ in the confining quark-gluon string.

The mass of bound state shows the minimum versus the constituent mass at $\mu_q^\star$, which gives the optimal value of mass for the calculation of radial wave function. For the constituent mass of strange quark we use

$$\mu_s = m_s + \mu_q^\star,$$

with $m_s = 0.24$ GeV, which represents the current mass at the scale of 1 GeV [13].

At this stage the estimates of coupling constants in the potential model can be optimally got according to (35) and (36). However, the corrections by both the quark-gluon loops and a relativistic motion can be rather essential, that can be taken into account by the introduction of $\mathcal{K}$-factor,

$$f = \mathcal{K}\, f^{\rm PM}.$$

At

$$\mathcal{K} = \frac{1}{1.45},$$

we get the estimates

$$\begin{aligned} f_\rho &= 205 \text{ MeV}, & (39)\\ f_{K^*} &= 217 \text{ MeV}, & (40)\\ f_{K_0^*} &= 130 \text{ MeV}. & (41) \end{aligned}$$

The $\mathcal{K}$-factor should generally depend on the spin and flavor of current under srudy. The above estimates show that the dependence on the flavors of quarks composing the bound state is rather suppressed, since we have amazingly reproduced the coupling constants of vector states in the limits of experimental intervals with the uniform $\mathcal{K}$-factor. As for the dependence on the quantum numbers of the meson, we expect that the variation of $\mathcal{K}$-factor is negligibly small because the summed quark spin in both K^* and K_0^* is equal to 1, while the spin-orbital contributions are usually suppressed. Thus, the estimate in (41) should be quite accurate up to 5 MeV, as it does for ρ and K^*. Nevertheless, we permit a conservative variation[4]

$$120\ \text{MeV} < f_{K_0^*} < 140\ \text{MeV}. \tag{42}$$

Further, we can compare the model estimates with the experimental data on K_{l3} decays listed in the Introduction. This analysis is presented in Figs. 5 and 6. We draw the conclusion on the model is well adjusted in describing the data.

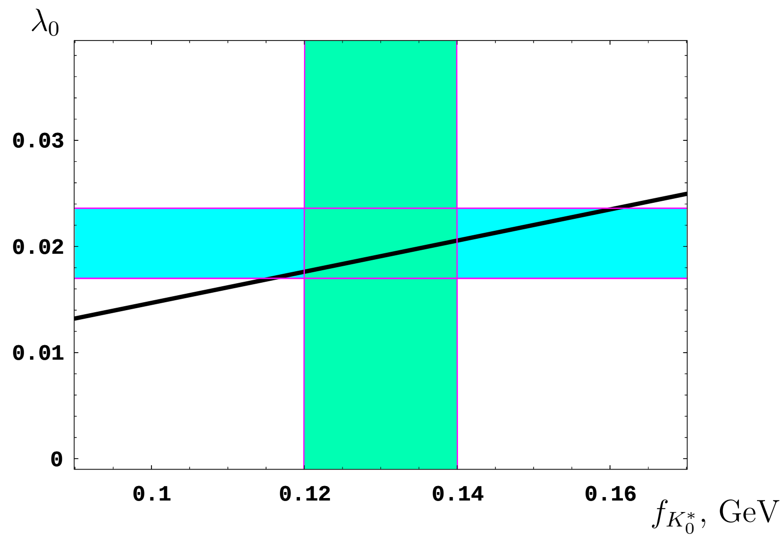

Figure 5: The model predictions for the slope λ_0 versus the coupling constant of K_0^* meson (the solid line) in comparison with the experimental data (the horizontal band). The vertical band gives the region of preferable values of $f_{K_0^*}$ expected from the potential model.

According to (29) and (32) the values of λ_+ and $f_+(0)$ are independent of $f_{K_0^*}$. Numerically, we get

$$\lambda_+ = 0.0271 \pm 0.0011, \quad f_+(0) = 1.046 \pm 0.040, \tag{43}$$

which are in a good agreement with both the experimental data and predictions of χPT.

Further, we test the model under the Callan–Treiman relation, that expresses the sum of vector-current form factors in terms of leptonic constants of kaon and pion:

$$f_+(m_K^2) + f_-(m_K^2) = \frac{f_K}{f_\pi}. \tag{44}$$

In the model under study we get

$$f_+(m_K^2) + f_-(m_K^2) = f_+(0) + \frac{g_{K_0^* K\pi} f_{K_0^*}}{m_{K_0^*}^2 - m_K^2} \approx f_+(0) + \frac{g_{K_0^* K\pi} f_{K_0^*}}{m_{K_0^*}^2}, \tag{45}$$

[4]See the sum rule estimates in [13].

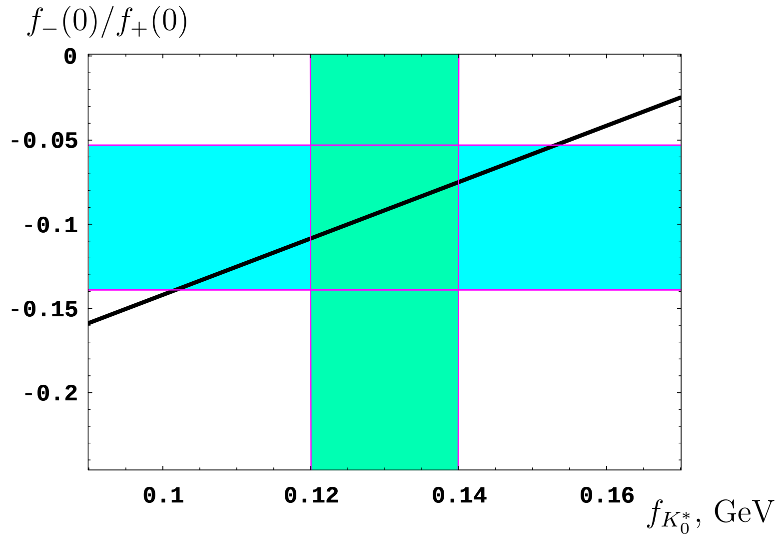

Figure 6: The model predictions for the ratio $f_-(0)/f_+(0)$ versus the coupling constant of K_0^* meson (the solid line) in comparison with the experimental data (the horizontal band). The vertical band gives the region of preferable values of $f_{K_0^*}$ expected from the potential model.

where we have neglected the kaon mass with respect to the scalar meson one. Then, numerically the relations result in

$$\frac{f_K}{f_\pi} = 1.305 \pm 0.020 \quad \text{or} \quad \frac{f_K}{f_\pi} \approx 1.273 \pm 0.017$$

under the variation in (42). So, at $f_\pi = 132$ MeV we deduce

$$f_K = 172 \pm 3 \text{ MeV} \quad \text{or} \quad f_K = 168 \pm 4 \text{ MeV}.$$

The systematic error caused by the approximation, as we see, is about 5 MeV, and conservatively one expects

$$f_K = 170 \pm 4 \pm 5 \text{ MeV},$$

which is in a good agreement with the known data.

Neglecting both the deviation of $f_+(0)$ from the unit and the kaon mass with respect to the mass of scalar K_0^*, we can derive from (44) and (45) the Dashen–Weinstein relation

$$\lambda_0 = \frac{m_\pi^2}{m_K^2 - m_\pi^2} \left(\frac{f_K}{f_\pi} - 1\right).$$

Both relations by Callan–Treiman and Dashen–Weinstein can acquire valuable numerical corrections in the model under study as well as in the χPT. So, from the formula for λ_0 we get

$$f_K = 160 \pm 4 \text{ MeV},$$

so that the displacement of f_K points to the possible size of corrections.

Then, we calculate the expression for the hadronic matrix element of tensor quark-current

$$\langle\pi^0(p_\pi)|\,\bar{s}\sigma_{\mu\nu}u\,|K^+(p_K)\rangle = -\mathrm{i}\,\frac{f_+(q^2)}{\sqrt{2}\,m_{K^*}}(p_\mu q_\nu - p_\nu q_\mu) \approx -\mathrm{i}\,\frac{f_+(0)}{\sqrt{2}\,m_{K^*}}(p_\mu q_\nu - p_\nu q_\mu), \tag{46}$$

where we have neglected the dependence of f_+ on q^2, since the antisymmetric tensor is linear in q. Formula (46) can be compared with the general expression

$$\langle\pi^0(p_\pi)|\,\bar{s}\sigma_{\mu\nu}u\,|K^+(p_K)\rangle = -\frac{\mathrm{i}}{\sqrt{2}}\,\mathcal{B}\,(p_\mu q_\nu - p_\nu q_\mu), \tag{47}$$

so that $\mathcal{B}$ depends on a single additional quantity $c_-(q^2)$

$$\mathcal{B} = c_-\,\frac{f_0(q^2)}{m_s - m_u} + (m_s + m_u)\,\frac{f_-(q^2)}{p\cdot q},$$

where we have explored the definition

$$\langle\pi^0(p_\pi)|\,\mathrm{i}\Big\{\bar{s}(\partial_\mu u) - (\partial_\mu\bar{s})u\Big\}\,|K^+(p_K)\rangle = (c_+\,p_\mu + c_-\,q_\mu)\,\langle\pi^0(p_\pi)|\,\bar{s}u\,|K^+(p_K)\rangle,$$

with an evident condition of self-consistency

$$c_+ = -\frac{1}{p\cdot q}\,(m_s^2 - m_u^2 + c_-\,q^2).$$

In the model of meson dominance we get

$$c_- = \frac{f_+}{f_0}\,\frac{m_s - m_u}{m_{K^*}} - \frac{f_-}{f_0}\,\frac{m_s^2 - m_u^2}{p\cdot q}. \tag{48}$$

Neglecting the current mass of light quark, at $q^2 = 0$ we find

$$c_-(0) \approx \frac{m_s}{m_{K^*}} + (\lambda_+ - \lambda_0)\,\frac{m_s^2}{m_\pi^2}, \tag{49}$$

$$c_+(0) \approx -\frac{m_s^2}{m_K^2 - m_\pi^2}. \tag{50}$$

The physical meaning of $c_\pm$ is rather simple: they determine the difference between the fractions of meson momenta carried by the $\bar{s}$ and u quarks in the kaon and pion under the weak transition. At $q^2 = 0$ we get

$$\alpha_K = \frac{1}{2}(c_+ + c_-) \approx 0.018, \tag{51}$$

$$\alpha_\pi = \frac{1}{2}(c_- - c_+) \approx 0.28. \tag{52}$$

4 Constraints on the leptoquark scales

Under the determination of hadronic matrix elements of quark currents we derive the ratios of form factors due to the contribution of leptoquark interactions,

$$\frac{f_S}{f_+(0)} = \frac{\sqrt{2}}{16G_{\rm F}|V_{su}|}\,\frac{m_K^2-m_\pi^2}{(m_s-m_u)m_K}\,\frac{1}{\Lambda_{LQ}^2}, \tag{53}$$

$$\frac{f_T}{f_+(0)} = -\frac{\sqrt{2}}{32G_{\rm F}|V_{su}|}\,\frac{m_K}{m_{K^*}}\,\frac{1}{\Lambda_{LQ}^2}, \tag{54}$$

where we have supposed the positive definiteness of Yukawa-constant products with respect to the mixing V_{su}. Then we extract the values of leptoquark scales in the tensor part,

$$\Lambda_{LQ} = 0.48^{+\infty}_{-0.17}\ \text{TeV}, \tag{55}$$

while the scalar form factor gives more stringent limit

$$\Lambda_{LQ} = 3.4^{+\infty}_{-1.1}\ \text{TeV}. \tag{56}$$

Thus, we deduce the 95%-confidence level

$$\Lambda_{LQ} > 1.2\ \text{TeV}.$$

Let us compare the above restriction on the parameters of leptoquark interactions with the constraints following from other processes relevant to the effective vertices induced by diagrams in Fig. 1. Since the Yukawa constants are flavor dependent, the direct constraints can be obtained from the leptonic decays of kaon, *viz.*, from both the electron and muon ones. In this way, the tensor interaction does not contribute, while the scalar one results in the multiplicative scaling of the decay amplitude. The factor has the form

$$\mathcal{K}_{LQ} \approx 1 - 2\,\frac{f_S}{f_+(0)}\,\frac{m_K}{m_l}, \tag{57}$$

where we have neglected the masses of pion and u-quark, and m_l denotes the mass of lepton.

The leptonic modes are measured with the accuracy of branching ratios

$$\frac{\delta\mathcal{B}_{\rm e}}{\mathcal{B}_{\rm e}} \approx \frac{1}{22}, \quad \frac{\delta\mathcal{B}_\mu}{\mathcal{B}_\mu} \approx \frac{1}{300},$$

that can be used in order to restrict the scalar interactions induced by the leptoquarks. So, taking the ratio of branching ratios, which is independent of both the leptonic constant of kaon and the CKM element $|V_{su}|$, we get the expression

$$\frac{\mathcal{B}_{\rm e}}{\mathcal{B}_\mu} = \frac{m_e^2}{m_\mu^2}\,\frac{1-4\,\dfrac{f_S}{f_+(0)}\,\dfrac{m_K}{m_e}}{1-4\,\dfrac{f_S}{f_+(0)}\,\dfrac{m_K}{m_\mu}} = 2.3372\cdot 10^{-5}\cdot\frac{1-4\,\dfrac{f_S}{f_+(0)}\,\dfrac{m_K}{m_e}}{1-4\,\dfrac{f_S}{f_+(0)}\,\dfrac{m_K}{m_\mu}},$$

where we expand in small corrections following from the leptoquark interactions. Comparing with the experimental result

$$\left.\frac{\mathcal{B}_{\rm e}}{\mathcal{B}_\mu}\right|_{\rm exp.} = (2.44 \pm 0.11) \cdot 10^{-5},$$

we find[5]

$$\Lambda_{LQ} > 43 \text{ TeV}.$$

Thus, the measurements of semileptonic kaon decay provide us with the soft confirmation of constraints following from the leptonic decays, since the tensor and scalar effective vertices correlate in the leptoquark interactions.

5 Discussion

In this paper we have developed a model of meson dominance, which has allowed us to get quite an accurate description of hadronic form factors in the decay $K^+ \to \pi^0 l^+ \nu_l$. In this way we have adjusted the model under the experimental data on the matrix element of vector quark-current and calculated the matrix element of tensor current induced by the leptoquark interactions. The experimental data on the semileptonic decay of kaon allow us to extract the constraints on the contributions beyond the Standard Model, so that

$$\Lambda_{LQ} > 1.2 \text{ TeV},$$

where Λ_{LQ} represents the ratio of leptoquark mass to the square of Yukawa-like coupling. This limit softly confirms the bounds following from the leptonic decays of kaon.

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